VIDYA BHAWAN BALIKA VIDYA PITH

शक्तिउत्थानआश्रमलखीसरायबिहार

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Positive Integral Powers of a Square Matrix

Let A be a square matrix. Then, we can define

1. $A^{n+1} = A^n . A$, where $n \in N$.
2. $A^m . A^n = A^{m+n}$
3. $(A^m)^n = A^{mn}$, $\forall$ m, $n \in N$

Matrix Polynomial

Let $f(x)= a_0x^n + a_1x^{n-1} -1 + a_2x^{n-2} + \ldots + a_n$. Then

$f(A)= a_0A^n + a_1A^{n-2} + \ldots + a_nI_n$

is called the matrix polynomial.

Transpose of a Matrix

Let $A = [a_{ij}]_{m \times n}$, be a matrix of order m x n. Then, the n x m matrix obtained by interchanging the rows and columns of A is called the transpose of A and is denoted by A' or A^T.

$A' = A^T = [a_{ij}]_{n \times m}$

Properties of Transpose

1. (A')' = A
2. (A + B)' = A' + B'
3. (AB)' = B'A'
4. (KA)' = kA'
5. $(A^N)' = (A')^N$
6. (ABC)' = C' B' A'

Symmetric and Skew-Symmetric Matrices

1. A square matrix $A = [a_{ij}]_{n \times n}$, is said to be symmetric, if A' = A.
   i.e., $a_{ij} = a_{ji}$, $\forall$i and j.
2. A square matrix A is said to be skew-symmetric matrices, if i.e., aij = — aji, di and j

Properties of Symmetric and Skew-Symmetric Matrices

1. Elements of principal diagonals of a skew-symmetric matrix are all zero. i.e., a_{ii} = — a_{ii} 2_{ii} = 0 or a_{ii} = 0, for all values of i.
2. If A is a square matrix, then
   (a) A + A' is symmetric.
   (b) A — A' is skew-symmetric matrix.
3. If A and B are two symmetric (or skew-symmetric) matrices of same order, then A + B is also symmetric (or skew-symmetric).
4. If A is symmetric (or skew-symmetric), then kA (k is a scalar) is also symmetric for skew-symmetric matrix.
5. If A and B are symmetric matrices of the same order, then the product AB is symmetric, iff BA = AB.

6. Every square matrix can be expressed uniquely as the sum of a symmetric and a skew-symmetric matrix.
7. The matrix B' AB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric matrix.
8. All positive integral powers of a symmetric matrix are symmetric.
9. All positive odd integral powers of a skew-symmetric matrix are skew-symmetric and positive even integral powers of a skew-symmetric are symmetric matrix.
10. If A and B are symmetric matrices of the same order, then
    (a) AB – BA is a skew-symmetric and
    (b) AB + BA is symmetric.
11. For a square matrix A, AA' and A' A are symmetric matrix.